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The 'colour polarigraph' – a simple method for determining the two-dimensional distribution of sugar concentration

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A method is described for photographically recording the two-dimensional distribution of sugar concentration in a tank, which typically yields a resolution of 0·01 in weight fraction sugar. The tank image is coloured according to the sugar concentration, giving a quantitative visual record of sugar contours. The technique is demonstrated in photographs of a laboratory model of an oceanic front, in which interleaving quasi-horizontal layers are formed by double-diffusive effects.

1. Introduction

Optical rotation, whereby the axis of linearly polarized light is rotated an amount proportional to the light path of optically active solution, is often used as a means of determining the concentration of sugars in solution (Washburn 1927). As discussed in Ruddick & Shirtcliffe (1979) for the case of two common laboratory solutes, sucrose and sodium chloride, optical rotation is the most direct measure of sugar concentration. It is almost linearly proportional to sugar concentration, and nearly unaffected by salt. Other physical properties such as refractive index and conductivity are strong functions of both sugar and salt concentration. Rotation is also insensitive to solution temperature, decreasing by roughly 0·3 % per degree Celsius increase.

In § 2 is described a simple and convenient method for photographically recording the two-dimensional distribution of sugar concentration, averaged across the third dimension of the tank. The method involves a form of optical polarimetry (measuring the amount by which linearly polarized light has been rotated by an optically active sugar solution), which is recorded photographically, so we call the technique the 'polarigraph'. Because the optical rotatory activity of a sugar solution is a strong monotonic function of the wavelength of light, the image of the experimental tank is colour coded according to the concentration of sugar. By using a calibration tank the method is made quantitative, with an estimated accuracy in weight fraction sugar (grams sugar per gram solution) of 0·01. The method is suitable for 'double-diffusive' laboratory studies (salt/sugar) as shown in the example in § 3, and can be used in laboratory studies involving a single solute (sugar).

2. The method

As shown in figure 1, the colour polarigraph apparatus is quite simple. A projector is set well back from the tank so as to evenly illuminate the diffusing screen (ground glass or tracing paper). Light then passes through the large sheet of Polaroid plastic polarizing film, through the experimental tank, through the polarizing

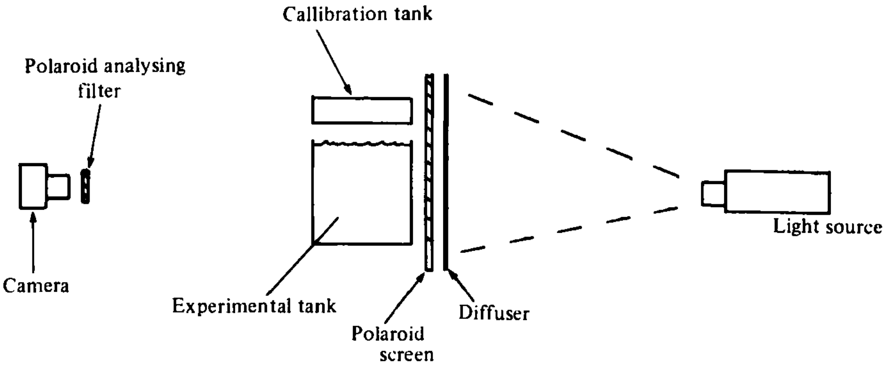

FIGURE 1. Sketch of the experimental set-up, showing the projector light source, diffuser, polaroid screen, experimental tank, calibration tank, polaroid analyzing filter, and camera.

filter on the camera lens, and into the camera.† It is important to have the projector and camera far enough back from the other apparatus to avoid uneven brightness in the image, which can cause false colour rendition on the film. A glass tank is preferred, since manufacturing stresses cause Perspex to be optically active, noticeably affecting the measurements.

The method is based on the fact that optical rotation is a strong monotonic function of the wavelength of light (Washburn 1927). Each wavelength component of white, linearly polarized light passing through the tank emerges with a different polarization angle. Predominantly one wavelength is stopped by the analyzer (the polaroid filter in the camera) because its angle of polarization is perpendicular to that of the analyser. The remaining light has a hue which is determined by the wavelength of light absorbed. The changes in colour correspond to contours of equal sugar concentration. The colour changes can be moved to correspond to particular sugar concentrations by rotating the analyser. By taking several photographs with the analyser in different orientations, a set of contours of sugar concentration can be built up.

For the large (2 m long) tank described in § 3 it was found necessary to have both projector and camera at least three metres back from the tank, and to use four sheets of diffusing paper, in order to provide even illumination of the tank image, and to reduce parallax errors. This, plus the fact that the 'crossed polaroids' absorbed well over half of the incident light, necessitated long exposures, about 10 s at $f/4$ for ASA 160 film. Long exposures caused no problem for the relatively slowly evolving phenomenon which was being studied. Rapidly evolving structures can easily be studied by using flash. In any case, it is advisable to try several test exposures to obtain the best colour saturation in a given experimental set-up.

To a good approximation the angle of rotation, ϕ, of polarized light is proportional to the product of optical path length, l, through the sugar solution at concentration W_S:

$$\phi = lW_S R(W_S, W_T, \theta, \lambda), \tag{1}$$

where W_T is the sodium chloride concentration in the solution, g salt per g solution, θ is the temperature of the solution, λ is the wavelength of the light. From equation (5.5) of Ruddick & Shirtcliffe (1979), at $\theta = 20$ °C and $\lambda = 589{\cdot}3\,\mu\text{m}$ (sodium light),

$$R = (6{\cdot}6326 + 2{\cdot}6451 W_S + 0{\cdot}7047 W_S^2)(1 + 0{\cdot}439 W_T). \tag{2}$$

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† John Taylor has recently discovered that by interchanging the light source and camera in figure 1, the polarigraph and shadowgraph can be combined.

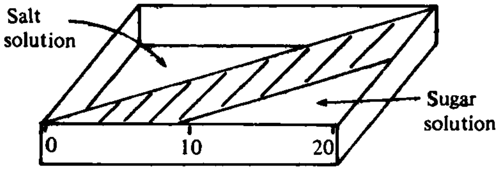

FIGURE 2. Details of the calibration tank. The numbers on the front correspond to path length in centimetres.

The 'calibration tank' shown in figure 2 is simply a small glass tank split diagonally into two triangular compartments. One compartment is filled with a sugar solution of the highest concentration used in the experiment. When placed in the field of view of the polarimetry apparatus, this 'wedge' of sugar solution presents a linearly increasing path length of sugar solution to the camera, which, provided W_S is sufficiently small, corresponds via equation (1) to an approximately linear increase in W_S at constant path length. The slight nonlinearity can easily be corrected by using equation (2) to make the calibration tank scale nonlinear. In this manner, the coloured bands are calibrated directly to sugar concentration. Since the calibration tank appears in every polarigraph, the problem of colour shift on the film due to exposure variation is overcome. To avoid refraction of light rays at the diagonal compartment boundary, the rear triangular compartment is filled with a (non-optically active) salt solution whose concentration is chosen so as to have the same refractive index as the sugar solution. The refractive indices can be matched with a refractometer, or by using a hydrometer to measure approximate densities and relating these to refractive index via equations (2.2), (2.4), (2.7) and (2.8) of Ruddick & Shirtcliffe (1979).

By tracing along the boundaries of the apparent colour changes in the projected image, contours of constant sugar concentration were drawn by hand. Three polarigraphs, taken with the analyser rotated so as to move the colour fringes to different locations, were used to build up the relatively complete set of contours in figure 5, to an estimated accuracy of 0·01 in W_S. This was 0·004 g cm^{-3} in density, or 4 % of full scale, with a tank breadth, l, of 20 cm. Increasing either l or the maximum sugar concentration will improve the resolution and accuracy proportionately, as the colour fringes will become more closely spaced.

The various systematic errors of the method are discussed in the appendix. So long as a sufficiently long lens (400 mm) is used on the camera, and the variation of R in (1) with W_S is corrected for (a trivial matter), the remaining systematic errors are about 5 % of full scale. An independent measurement of the conductivity or refractive index field would be necessary to correct for these.

Use of an automated scanning colour analyser would reduce the random error of 0·01 in W_S due to human colour perception by a factor of 5 or 10. In order for this level of sophistication to be practical, it would be necessary to simultaneously measure refractive index or conductivity and correct for the sources of systematic error discussed in the Appendix.

An alternative method, tested in principle but not actually used, results in a single contour on a black and white film. If the projector is replaced by a bright monochromatic light source, and a fast black and white film is greatly overexposed in the set-up in figure 1, only the regions of the tank corresponding to the 'crossed polaroids' null concentration for the wavelength used will be unexposed. This will leave a dark line on the negative corresponding to a single contour of sugar concentration.

3. An example of its use

The technique is illustrated with a laboratory experiment used to study double-diffusively driven interleaving at a thermohaline front (Ruddick & Turner 1979), across which there is a large temperature/salt anomaly. A tank is filled with a linear density gradient of sugar solution (analogous to warm, salty water) on one side of a removable vertical barrier, and the same density gradient of salt solution (analogous to cold, fresh water) on the other side. Figure 3 (*a*) (plate 1) is a colour polarigraph of the tank before the barrier is removed, showing the undisturbed sugar gradient on the left. The contours are not perfectly horizontal because of a parallax effect, discussed in the appendix.

After the barrier is withdrawn, internal wave motions arise due to small imbalances in the density structure, but these soon die away, leaving a slightly distorted but essentially vertical front. After some time, a regular series of interleaving layers forms, and each layer steadily extends into undisturbed fluid.

Figure 3 (*b*) (plate 1) is a colour polarigraph of the layers, showing that layers of sugar water are intruding toward the right and that salt water (the clear regions) is intruding toward the left. The distortions of the dye streaks show the horizontal motions responsible for the convection, and the correlation in depth between velocity and sugar concentration shows that a net horizontal convective sugar flux is occurring.

The structure of the interleaving layers is shown by a shadowgraph, figure 4, in which the refractive index gradients in the tank focus projected light on a tracing-paper screen. It is seen that each layer consists of a region of sugar fingers, formed by the presence of sugar water over salt water, with a diffusive interface above and below. (See Turner 1973 for a description of fingers and diffusive interfaces.) As the sugar water is advected to the right, the finger fluxes cause it to become less dense and rise. This causes the tilt in the layers.

Figure 5 shows a set of contours of sugar concentration derived from figure 3 (*b*) and two other polarigraphs taken with the analyzer on the camera rotated to various orientations. With little effort, a fairly complete description of the sugar field can be built up. It is seen that the 'average' sugar contours have spread with the layers, in a manner consistent with the convective sugar flux.

Appendix. Accuracy of the method

As discussed earlier, hand analysis of the polarigraphs results in a random error of about 0·01 in weight fraction sugar (typically 4 % of full scale). Automated scanning would reduce this considerably but, in order to be practical, the systematic errors discussed below would need to be corrected, and that would require a separate determination of the salinity field.

The variation of optical activity (the coefficient R in equation (1)) with W_S leads to a difference of 0·005 in W_S between the linearly spaced calibration tank fringes and those in the experimental tank. At a typical full scale W_S of 0·26 (density 1·1 g cm^{-3}), this is a 2 % error. This can be corrected for (and was for figure 5) using equation (2).

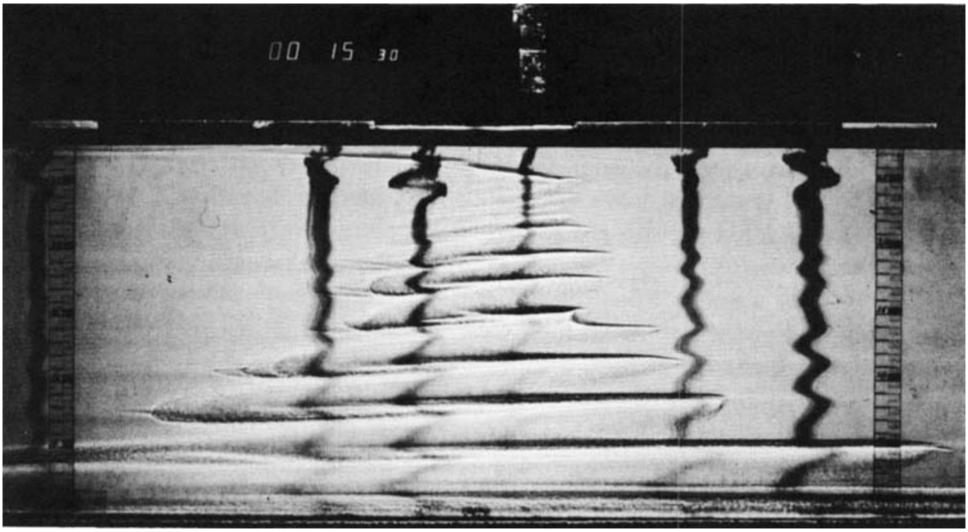

FIGURE 4. Shadowgraph of the experimental tank 15 min 30 s after removal of the barrier. The dyelines were introduced at about 15 min 10 s. The heavy black marks are at 5 cm intervals.

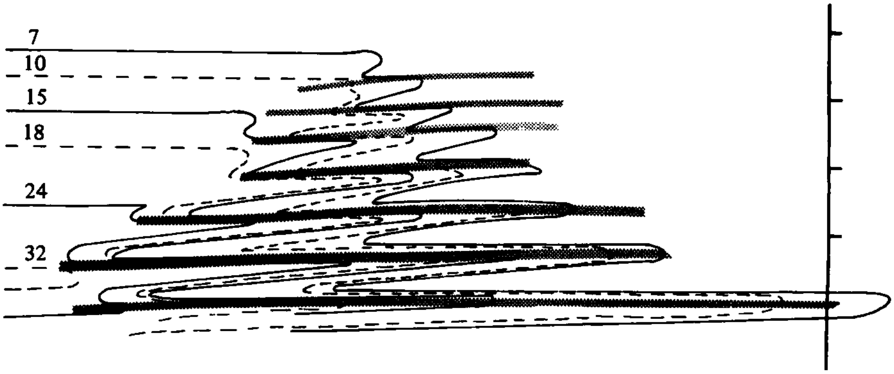

FIGURE 5. Contours of sugar concentration in parts per thousand, at time 17 min 30 s ± 30 s. The contours have been drawn by hand from a series of polarigraphs, one of which is figure 3(*b*), based on the transitions from one hue to the next. A scale marked in 5 cm increments appears to the right. The diffusive interfaces, as deduced from figure 4, are shown as stippled regions.

The effect of salt in solution is to reduce the optical rotation according to equation (2). At 10 % density of salt solution ($W_T = 0{\cdot}14$), R is reduced by 6 %. In practice, the error is only 3 % of full scale, since full salt concentration occurs at zero sugar concentration and vice versa. An independent measurement of the field of some other property such as conductivity would be necessary to correct for this effect.

So long as the temperature is uniform in both the calibration and experimental tanks, the effect of temperature variation on optical activity is automatically compensated for. In practice, this variation is so small ($R^{-1}dR/d\theta \simeq 0{\cdot}003$ per °C) that variations of several degrees can be tolerated.

Variations in sugar and salt concentration in the experimental tank cause refractive index gradients which can deflect light rays passing through the tank. In the example shown in figures 3–5, this deflection was as large as ± 0·5 cm. The error in determining the accuracy of the position in the tank represented by a given measurement resulted in relative displacement of the sugar inversion regions deduced by the polarigraph, and the positions of the diffusive interfaces as deduced from shadowgraph. The former are displaced upward, while the latter (shown in figure 5 as dotted regions) are displaced downward. The combined displacements are as large as 1 cm in this example. The additional path length traversed by the deflected ray in the polarigraph leads to an error of a few percent of full-scale sugar concentration. Unless the refractive index gradients were known (perhaps through schlieren photography) for the region, the error would be a difficult one to correct.

One course of error which could have been minimized had its severity been recognized was parallax due to using too short a camera lens. For the 100 mm lens used, the field of view at the left or right of the image is about 12° off the lens axis. Thus the path length through the tank is about $1/\cos 12° = 1{\cdot}022$ times the path, a 2·2 % error. This effect caused the slopes of the colour fringes in figure 3(*a*). The use of a 200 mm lens would have reduced this error to 0·4 %. The 12° angle also caused the averaging to occur over a ± 2 cm horizontal extent. This is unacceptable where horizontal gradients are large, as at the noses of the interleaving layers. Setting the camera back and using a 400 mm lens would reduce this error to acceptable levels.

In summary, so long as a long lens is used to keep parallax errors to acceptable levels, typical error levels are:

(*a*) reading (by hand), ± 4 %;
(*b*) variation of R with W_S, 2 %;
(*c*) variation of R with W_T, 3 %;
(*d*) deflection of light rays ½ cm displacement, 2 %.

Error (*b*) can be corrected for trivially, leaving 4 % random and 5 % systematic error in the 'read by hand' system. If the reading is automated, it is conceivable that (*a*) can be reduced to 1 % or less. If there is no salt, then error (*c*) is zero and (*d*) can be corrected for. If salt is present, an independent measurement such as conductivity or schlieren photography to obtain the refractive index field, would be necessary to correct for (*c*) and (*d*).

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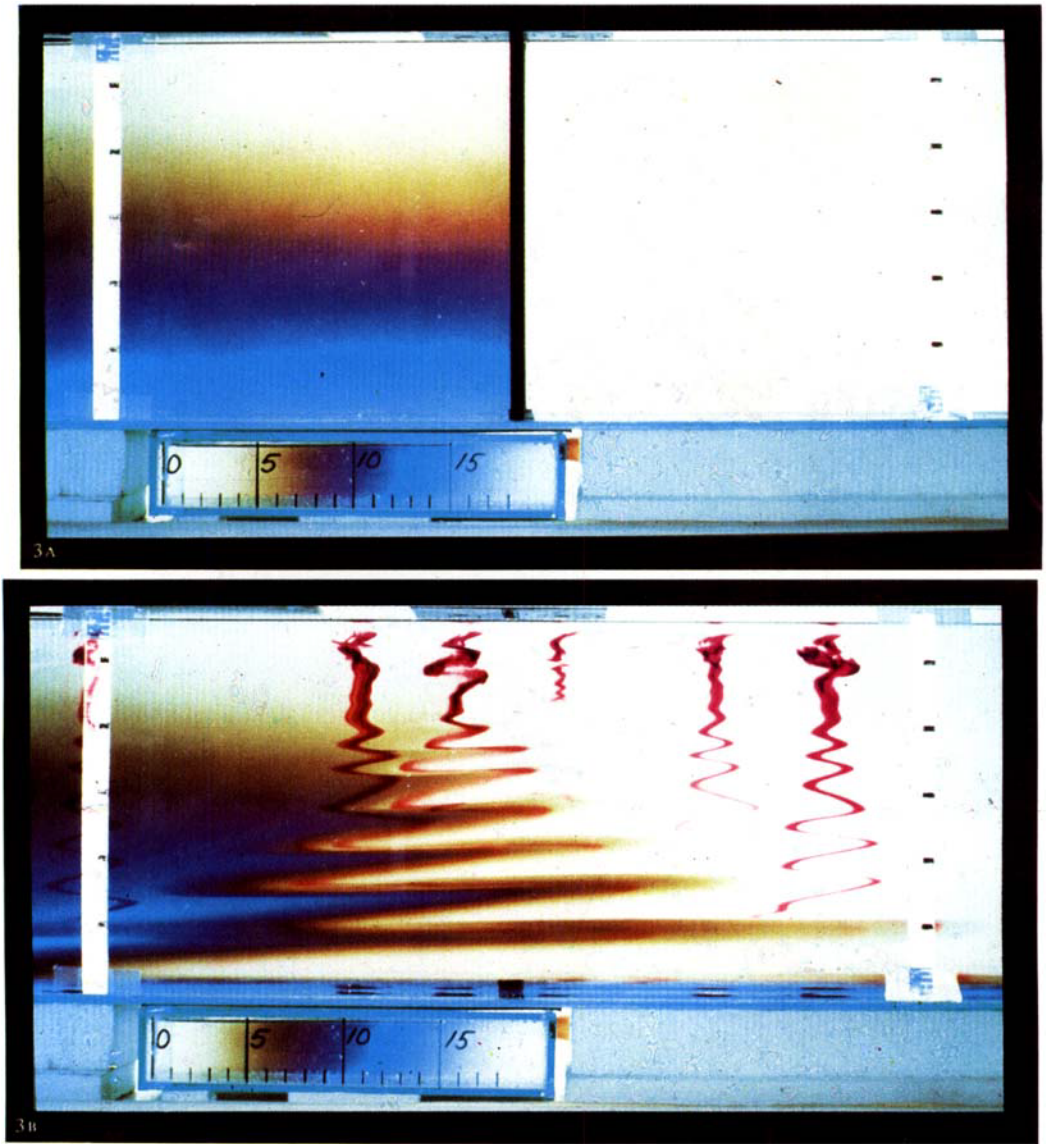

Figure 3. (*a*) Colour polarimeter photograph of the experimental tank, with stratified sugar solution on the left and stratified salt solution on the right, separated by a removable vertical barrier. The black marks on the scales at the right and left of the tank are at 5 cm intervals. The ends of the tank are well beyond the field of view of the camera. The calibration tank is visible at the bottom; the numbers 0, 5, 10, 15 correspond to optical path length of sugar solution, in centimetres. (*b*) Colour polarimeter photograph of the experimental tank 17 min 20 s after the barrier was removed, showing how the sugar solution has flowed to the right in a regular series of interleaving layers. The distorted purple dyelines were introduced at 15 min 10 s, and indicate the velocity shears.